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CONTENT

String and Brane Tensions as Dynamical Degrees of Freedom

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Abstract

We discuss a new class of string and p-brane models where the string/brane tension appears as an *additional dynamical degree of freedom* instead of being introduced by hand as an *ad hoc* dimensionfull scale. The latter property turns out to have a significant impact on the string/brane dynamics. The dynamical tension obeys Maxwell (or Yang-Mills) equations of motion (in the string case) or their rank p gauge theory analogues (in the p-brane case), which in particular triggers a simple classical mechanism of ("color") charge confinement.

Keywords: modified string and p-brane actions, reparametrization-covariant integration measures, dynamical generation of string/brane tension, color charge confinement.

1 Introduction

In order to build actions describing dynamics in geometrically motivated field theories (for reviews of string and brane theories, see [1]) we need among other things a consistent generally-covariant integration measure density, *i.e.*, covariant under arbitrary diffeomorphisms (reparametrizations) on the underlying space-time manifold. Usually the natural choice is the standard Riemannian metric density $\sqrt{-g}$ with $g \equiv \det\|g_{\mu\nu}\|$. However, there are no purely geometric reasons which prevent us from employing an alternative generally-covariant integration measure. For instance, introducing additional D scalar fields φ^i ($i = 1, \ldots, D$ where D is the space-time dimension) we may take the following new non-Riemannian measure density $\Phi(\varphi)$:

$$\Phi(\varphi) \equiv \frac{1}{D!}\varepsilon^{\mu_1\ldots\mu_D}\varepsilon_{i_1\ldots i_D}\partial_{\mu_1}\varphi^{i_1}\ldots\partial_{\mu_D}\varphi^{i_D} \; . \tag{1}$$

Using (1) allows to construct new classes of models involving Gravity called *Two-Measure Gravitational Models* [2], whose actions are typically of the form:

$$S = \int d^D x\, \Phi(\varphi)\, L_1 + \int d^D x\, \sqrt{-g}\, L_2 \; , \tag{2}$$

$$L_{1,2} = e^{\frac{\alpha\phi}{M_P}}\Big[-\frac{1}{\kappa}R(g,\Gamma) - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \big(\text{Higgs}\big) + \big(\text{fermions}\big)\Big] \; . \tag{3}$$

where $R(g,\Gamma)$ is the scalar curvature in the first-order formalism (*i.e.*, the affine connection Γ is independent of the metric), ϕ is the dilaton field, M_P is the Planck mass, *etc.*. Although naively the additional "measure-density" scalars φ^i appear in (2) as pure-gauge degrees of freedom (due to the invariance under arbitrary diffeomorphisms in the φ^i-target space), there is a remnant – the so called "geometric" field $\zeta(x) \equiv \frac{\Phi(\varphi)}{\sqrt{-g}}$, which remains an additional dynamical degree of freedom beyond the standard physical degrees of freedom characteristic to the ordinary gravity models with the standard Riemannian-metric integration measure. The most important property of the "geometric" field $\zeta(x)$ is that its dynamics is determined only through the matter fields locally (*i.e.*, without gravitational interaction). The latter turns out to have a significant impact on the physical properties of the two-measure gravity models which allows them to address various basic problems in cosmology and particle physics phenomenology and provide physically plausible solutions, for instance: (i) the issue of scale invariance and its dynamical breakdown, *i.e.*, spontaneous generation of dimensionfull fundamental scales; (ii) cosmological constant problem; (iii) geometric origin of fermionic families.

For a recent review of two-measure gravity models see the contribution in this volume [3]. In what follows we are going to apply the above ideas to the case of string and p-brane models. Part of our exposition is based on earlier works [4]. Furthermore, we will elaborate on various important properties of the modified-measure string and brane models with dynamical string/brane tension.

2 Bosonic Strings with a Modified World-Sheet Integration Measure

We begin by first recalling the standard Polyakov-type action for the bosonic string [5]:

$$S_{\rm Pol} = -T \int d^2\sigma\, \frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X)\ . \tag{4}$$

Here $(\sigma^0,\sigma^1) \equiv (\tau,\sigma)$; $a,b = 0,1$; $\mu,\nu = 0,1,\ldots,D-1$; $G_{\mu\nu}$ denotes the Riemannian metric on the embedding space-time; γ_{ab} is the intrinsic Riemannian metric on the $1+1$-dimensional string world-sheet and $\gamma = \det\|\gamma_{ab}\|$; T indicates the string tension – a dimensionfull scale introduced *ad hoc*. The resulting equations of motion w.r.t. γ^{ab} and X^μ read, respectively:

$$T_{ab} \equiv \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu\right) G_{\mu\nu}(X) = 0\ , \tag{5}$$

$$\frac{1}{\sqrt{-\gamma}}\partial_a\left(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0\ , \tag{6}$$

where $\Gamma^\mu_{\nu\lambda} = \frac{1}{2}G^{\mu\kappa}\left(\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda}\right)$ is the affine connection for the external metric.

Let us now introduce two additional world-sheet scalar fields φ^i $(i = 1,2)$ and replace $\sqrt{-\gamma}$ with a new reparametrization-covariant world-sheet integration measure density $\Phi(\varphi)$ defined in terms of φ^i :

$$\Phi(\varphi) \equiv \frac{1}{2}\varepsilon_{ij}\varepsilon^{ab}\partial_a\varphi^i\partial_b\varphi^j = \varepsilon_{ij}\dot{\varphi}^i\partial_\sigma\varphi^j\ . \tag{7}$$

However, the naively generalized string action $S_1 = -\frac{1}{2}\int d^2\sigma\, \Phi(\varphi)\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X)$ has a problem: the equations of motion w.r.t. γ^{ab} lead to an unacceptable condition $\Phi(\varphi)\,\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) = 0$, *i.e.*, vanishing of the induced metric on the world-sheet.

To remedy the above situation let us consider topological (total-derivative) terms w.r.t. standard Riemannian world-sheet integration measure. Upon measure replacement $\sqrt{-\gamma} \to \Phi(\varphi)$ the former are *not any more* topological – they will contribute nontrivially to the equations of motion. For instance:

$$\int d^2\sigma\,\sqrt{-\gamma}\,R \to \int d^2\sigma\,\Phi(\varphi)\,R \quad , \quad R = \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}}\left(\partial_a\omega_b - \partial_b\omega_a\right)\ , \tag{8}$$

where R is the scalar curvature w.r.t. $D=2$ spin-connection $\omega_a^{\bar a\bar b} = \omega_a\varepsilon^{\bar a\bar b}$ (here $\bar a,\bar b$ denote tangent space indices). The vector field ω_a behaves as world-sheet Abelian gauge field.

Eq.(8) prompts us to construct the following consistent modified bosonic string action[1] :

$$S = -\int d^2\sigma\,\Phi(\varphi)\Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}}F_{ab}(A)\Big]\ , \tag{9}$$

[1] In ref.[6] another interesting geometric modification of the standard bosonic string model has been proposed, which is based on dynamical world-sheet metric and torsion.

where $\Phi(\varphi)$ is given by (7) and $F_{ab}(A) \equiv \partial_a A_b - \partial_b A_a$ is the field-strength of an auxiliary Abelian gauge field A_a. The action (9) is reparametrization-invariant as its ordinary string analogue (4). Furthermore, (9) is invariant under diffeomorphisms in φ-target space supplemented with a special conformal transformation of γ_{ab} :

$$\varphi^i \longrightarrow \varphi'^i = \varphi'^i(\varphi) \quad , \quad \gamma_{ab} \longrightarrow \gamma'_{ab} = J\gamma_{ab} \;\; , \;\; J \equiv \det\Bigl\| \frac{\partial \varphi'^i}{\partial \varphi^j} \Bigr\| \; . \tag{10}$$

The latter symmetry, which we will call *"Φ-extended Weyl symmetry"*, is the counterpart of the ordinary Weyl conformal symmetry of the standard string action (4).

The equations of motion w.r.t. φ^i resulting from (9) :

$$\varepsilon^{ab}\partial_b\varphi^i\partial_a\Bigl(\gamma^{cd}\partial_c X^\mu \partial_d X^\nu G_{\mu\nu}(X) - \frac{\varepsilon^{cd}}{\sqrt{-\gamma}}F_{cd}\Bigr) = 0 \tag{11}$$

imply (provided $\Phi(\varphi) \neq 0$) :

$$\gamma^{cd}\partial_c X^\mu \partial_d X^\nu G_{\mu\nu}(X) - \frac{\varepsilon^{cd}}{\sqrt{-\gamma}}F_{cd} = M \; \Bigl(= \mathrm{const}\Bigr) \; . \tag{12}$$

The equations of motion w.r.t. γ^{ab} are:

$$T_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - \frac{1}{2}\gamma_{ab}\frac{\varepsilon^{cd}}{\sqrt{-\gamma}}F_{cd} = 0 \; . \tag{13}$$

Both Eqs.(12)–(13) yield $M = 0$ and $\Bigl(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu\Bigr) G_{\mu\nu}(X) = 0$, which is the same as in standard Polyakov-type formulation (5).

The equations of motion w.r.t. X^μ read:

$$\partial_a\left(\Phi\gamma^{ab}\partial_b X^\mu\right) + \Phi\gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0 \; , \tag{14}$$

where again $\Gamma^\mu_{\nu\lambda}$ is the affine connection corresponding to the external space-time metric $G_{\mu\nu}$ as in the standard string case (6).

Most importantly, the equations of motion w.r.t. A_a resulting from (9) yield:

$$\varepsilon^{ab}\partial_b\Bigl(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\Bigr) = 0 \; , \tag{15}$$

which can be integrated to yield a *spontaneously induced* string tension:

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} = \mathrm{const} \equiv T \; .$$

Since the modified-measure string model (9) naturally requires the presence of the auxiliary Abelian world-sheet gauge field A_a, we may extend it by introducing a coupling of A_a to some world-sheet charge current j^a :

$$S = -\int d^2\sigma\, \Phi(\varphi)\Bigl[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}}F_{ab}(A)\Bigr] + \int A_a j^a \; . \tag{16}$$

In particular, we may take j^a to be the current of point-like charges on the string, so that in the "static" gauge:

$$\int A_a j^a = -\sum_i e_i \int d\tau A_0(\tau,\sigma_i) \; , \tag{17}$$

where σ_i $(0 < \sigma_1 < \ldots < \sigma_N \leq 2\pi)$ are the locations of the charges. Now, the action (16) produces A_a-equations of motion:

$$\varepsilon^{ab}\partial_b E + j^a = 0 \quad , \quad E \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \; . \tag{18}$$

Eqs.(18) look exactly as $D = 2$ Maxwell equations where the *variable* dynamical string tension $E \equiv \Phi(\varphi)/\sqrt{-\gamma}$ is identified as world-sheet electric field strength, *i.e.*, canonically conjugated momentum w.r.t. A_1 (the latter fact can be directly verified from the explicit form of the A-term in the action (9) or (16)).

Remark on Canonical Hamiltonian Treatment. Introducing the canonical momenta resulting from the action (16) :

$$\pi_i^\varphi = -\varepsilon_{ij}\partial_\sigma\varphi^j\Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu\partial_b X^\nu G_{\mu\nu} - \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}}F_{ab}(A)\Big] \; , \tag{19}$$

$$\pi_{A_1} \equiv E = \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \;\; , \;\; \mathcal{P}_\mu = -\Phi(\varphi)\left(\gamma^{00}\dot{X}^\nu + \gamma^{01}\partial_\sigma X^\nu\right) G_{\mu\nu} \; , \tag{20}$$

we obtain the canonical Hamiltonian as a linear combination of first-class constraints only. Part of the latter resemble the constraints in the ordinary string case $\pi_{\gamma^{ab}} = 0$ and

$$\mathcal{T}_\pm \equiv \frac{1}{4}G^{\mu\nu}\Big(\frac{\mathcal{P}_\mu}{E} \pm G_{\mu\kappa}\partial_\sigma X^\kappa\Big)\Big(\frac{\mathcal{P}_\nu}{E} \pm G_{\nu\lambda}\partial_\sigma X^\lambda\Big) = 0 \; ,$$

where in the last Virasoro constraints the dynamical string tension E appears instead of the *ad hoc* constant tension.

The rest of the Hamiltonian constraints are $\pi_{A_0} = 0$ and

$$\partial_\sigma E - \sum_i e_i\delta(\sigma - \sigma_i) = 0 \; , \tag{21}$$

i.e., the $D = 2$ "Gauss law" constraint for the dynamical string tension, which coincides with the 0-th component of Eq.(18). Finally, we have constraints involving only the measure-density fields:

$$\partial_\sigma\varphi^i\pi_i^\varphi = 0 \quad , \quad \frac{\pi_2^\varphi}{\partial_\sigma\varphi^1} = 0 \; . \tag{22}$$

The last two constraints span a closed Poisson-bracket algebra:

$$\Big\{\, \partial_\sigma\varphi^i\pi_i^\varphi(\sigma)\, ,\, \partial_{\sigma'}\varphi^i\pi_i^\varphi(\sigma')\,\Big\} = 2\partial_\sigma\varphi^i\pi_i^\varphi(\sigma)\partial_\sigma\delta(\sigma - \sigma') + \partial_\sigma\left(\partial_\sigma\varphi^i\pi_i^\varphi\right)\delta(\sigma - \sigma') \; ,$$

(a centerless Virasoro algebra), and:

$$\Big\{\, \partial_\sigma\varphi^i\pi_i^\varphi(\sigma)\, ,\, \frac{\pi_2^\varphi}{\partial_\sigma\varphi^1}(\sigma')\,\Big\} = -\partial_\sigma\Big(\frac{\pi_2^\varphi}{\partial_\sigma\varphi^1}\Big)\delta(\sigma - \sigma') \; .$$

Therefore, the constraints (22) imply that the measure-density scalars φ^i are pure-gauge degrees of freedom.

3 Classical Confinement Mechanism of "Color" Charges via Dynamical String Tension

3.1 Non-Abelian Generalization

First, let us notice the following identity in $D = 2$ involving Abelian gauge field A_a:

$$\frac{1}{2\sqrt{-\gamma}}\varepsilon^{ab}F_{ab}(A) = \sqrt{\frac{1}{2}F_{ab}(A)F_{cd}(A)\gamma^{ac}\gamma^{bd}} \; . \tag{23}$$

This suggests the proper extension of the modified-measure bosonic string model (9) by introducing a *non-Abelian* (*e.g.*, $SU(\mathcal{N})$) auxiliary gauge field A_a (here we take for simplicity flat external metric $G_{\mu\nu} = \eta_{\mu\nu}$) :

$$S = -\int d^2\sigma\,\Phi(\varphi)\Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu\partial_b X_\mu - \sqrt{\frac{1}{2}\,\mathrm{Tr}(F_{ab}(A)F_{cd}(A))\gamma^{ac}\gamma^{bd}}\Big]$$
$$= -\int d^2\sigma\,\Phi(\varphi)\Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu\partial_b X_\mu - \frac{1}{\sqrt{-\gamma}}\sqrt{\mathrm{Tr}(F_{01}(A)F_{01}(A))}\Big] \; , \tag{24}$$

where $F_{ab}(A) = \partial_a A_b - \partial_b A_c + i\left[A_a,\, A_b\right]$.

The action (24) is again invariant under the Φ-*extended Weyl (conformal)* symmetry (10).

Notice that the *"square-root" Yang-Mills* action (with the regular Riemannian-metric integration measure):

$$\int d^2\sigma\, \sqrt{-\gamma}\sqrt{\frac{1}{2}\,\mathrm{Tr}(F_{ab}(A)F_{cd}(A))\gamma^{ac}\gamma^{bd}} = \int d^2\sigma\, \sqrt{\mathrm{Tr}(F_{01}(A)F_{01}(A))} \tag{25}$$

is a "topological" action similarly to the $D = 3$ Chern-Simmons action (*i.e.*, it is metric-independent).

Similarly to the Abelian case (16) we can also add a coupling of the auxiliary non-Abelian gauge field A_a to an external "color"-charge world-sheet current j^a:

$$S = -\int d^2\sigma\, \Phi(\varphi)\Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu - \frac{1}{\sqrt{-\gamma}}\sqrt{\mathrm{Tr}(F_{01}(A)F_{01}(A))}\Big] + \int \mathrm{Tr}\,(A_a j^a)\ . \tag{26}$$

In particular, for a current of "color" point-like charges on the world-sheet in the "static" gauge :

$$\int \mathrm{Tr}\,(A_a j^a) = -\sum_i \mathrm{Tr}\, C_i \int d\tau A_0(\tau,\sigma_i)\ , \tag{27}$$

where σ_i $(0 < \sigma_1 < \ldots < \sigma_N \leq 2\pi)$ are the locations of the charges.

The action (26) produces the following equations of motion w.r.t. φ^i and γ^{ab}, respectively:

$$\frac{1}{2}\gamma^{cd}\partial_c X^\mu \partial_d X_\mu - \frac{1}{\sqrt{-\gamma}}\sqrt{\mathrm{Tr}(F_{01}F_{01})} = M\ \Big(= \mathrm{const}\Big)\ , \tag{28}$$

$$T_{ab} \equiv \partial_a X^\mu \partial_b X_\mu - \frac{1}{\sqrt{-\gamma}}\gamma_{ab}\sqrt{\mathrm{Tr}(F_{01}F_{01})} = 0\ . \tag{29}$$

As in the Abelian case the above Eqs.(28)–(29) imply $M = 0$ and the Polyakov-type equation (5).

The equations of motion w.r.t. auxiliary gauge field A_a resulting from (26) resemble, similarly to the Abelian case (18), the $D = 2$ non-Abelian Yang-Mills equations:

$$\varepsilon^{ab}\nabla_b \mathcal{E} + j^a = 0\ , \tag{30}$$

where:

$$\nabla_a \mathcal{E} \equiv \partial_a \mathcal{E} + i\left[A_a,\, \mathcal{E}\right] \quad , \quad \mathcal{E} \equiv \pi_{A_1} \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}\frac{F_{01}}{\sqrt{\mathrm{Tr}(F_{01}F_{01})}}\ . \tag{31}$$

Here $\mathcal{E}$ is the non-Abelian electric field-strength – the canonically conjugated momentum π_{A_1} of A_1, whose norm is the dynamical string tension $T \equiv |\mathcal{E}| = \Phi(\varphi)/\sqrt{-\gamma}$.

The equations of motion for the dynamical string tension following from (30) is:

$$\partial_a\Big(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\Big) + \varepsilon_{ab}\frac{\mathrm{Tr}\,\big(F_{01}j^b\big)}{\sqrt{\mathrm{Tr}\,(F_{01}^2)}} = 0\ . \tag{32}$$

In particular, the absence of external charges $(j^a = 0)$: $T \equiv \Phi(\varphi)/\sqrt{-\gamma} = T_0 \equiv \mathrm{const}$

Finally, the X^μ-equations of motion $\partial_a\left(\Phi(\varphi)\gamma^{ab}\partial_b X^\mu\right) = 0$ resulting from the action (26) can be rewritten in the conformal gauge $\sqrt{-\gamma}\gamma^{ab} = \eta^{ab}$ as:

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\partial^a\partial_a X^\mu - \widetilde{j}^a\partial_a X^\mu = 0 \quad , \ \text{where}\ \widetilde{j}_a \equiv \varepsilon_{ab}\frac{\mathrm{Tr}\,\big(F_{01}j^b\big)}{\sqrt{\mathrm{Tr}\,(F_{01}^2)}}\ . \tag{33}$$

For static charges $\widetilde{j}_1 = -\sum_i \widetilde{e}_i \delta(\sigma - \sigma_i)$:

$$T \equiv \Phi(\varphi)/\sqrt{-\gamma} = T_0 + \sum_i \widetilde{e}_i\theta(\sigma - \sigma_i) \quad , \quad \widetilde{e}_i \equiv \frac{\mathrm{Tr}\,(F_{01}C_i)}{\sqrt{\mathrm{Tr}\,(F_{01}^2)}}\Bigg|_{\sigma=\sigma_i}\ ; \tag{34}$$

$$T\partial^a\partial_a X^\mu + \Big(\sum_i \widetilde{e}_i\delta(\sigma - \sigma_i)\Big)\partial_\sigma X^\mu = 0 \ \rightarrow \begin{cases} \partial^a\partial_a X^\mu = 0 \\ \partial_\sigma X^\mu\Big|_{\sigma=\sigma_i} = 0 \end{cases}\ . \tag{35}$$

3.2 Classical Confinement Mechanism

Recall that the modified string action (26) yields the $D = 2$ Yang-Mills-like Eqs.(30) whose 0-th component $\partial_\sigma \mathcal{E} + i\Big[A_1\, ,\, \mathcal{E}\Big] + j^0 = 0$ is the "Gauss law" constraint for the dynamical string tension ($T \equiv |\mathcal{E}| = \Phi(\varphi)/\sqrt{-\gamma}$). For point-like "color" charges and taking the gauge $A_1 = 0$ (*i.e.*, $\mathcal{E} \to \widetilde{\mathcal{E}} = G\mathcal{E}G^{-1}$ where $A_1 = -iG^{-1}\partial_\sigma G$), the latter reads:

$$\partial_\sigma \widetilde{\mathcal{E}} - \sum_i \widetilde{C}_i \delta(\sigma - \sigma_i) = 0 \quad , \quad \widetilde{C}_i \equiv G C_i G^{-1} \Big|_{\sigma=\sigma_i} \; . \tag{36}$$

Let us consider the case of *closed* modified string with positions of the "color" charges at $0 < \sigma_1 < \ldots < \sigma_N \leq 2\pi$. Then, integrating the "Gauss law" constraint (36) along the string (at fixed proper time) we obtain:

$$\sum_i \widetilde{C}_i = 0 \quad , \quad \widetilde{\mathcal{E}}_{i,i+1} = \widetilde{\mathcal{E}}_{i-1,i} + \widetilde{C}_i \; , \tag{37}$$

where $\widetilde{\mathcal{E}}_{i,i+1} = \widetilde{\mathcal{E}}$ in the interval $\sigma_i < \sigma < \sigma_{i+1}$.

The discussion in this section leads to the following conclusions:

- We see from Eqs.(34)–(35) that the modified-measure (closed) string with N point-like ("color") charges on it ((16) or (26)) is equivalent to N chain-wise connected regular open string segments obeying Neumann boundary conditions.
- Each of the above open string segments, with end-points at the charges e_i and e_{i+1} (in the Abelian case) or C_i and C_{i+1} (in the non-Abelian case), has *different* constant string tension $T_{i,i+1}$ such that $T_{i,i+1} = T_{i-1,i} + \overset{(\sim)}{e}_i$ (the non-Abelian $\widetilde{e}_i$ are defined in (34)).
- Eq.(37) tells us that the only (classically) admissable configuration of "color" point-like charges coupled to a modified-measure closed bosonic string is the one with *zero* total "color" charge, *i.e.*, the model (26) provides a classical mechanism of "color" charge confinement.

4 Branes with a Modified World-Volume Integration Measure

Before generalizing our construction from the previous two sections to the case of higher-dimensional p-branes, let us recall the standard Polyakov-type formulation of the bosonic p-brane action:

$$S = -\frac{T}{2} \int d^{p+1}\sigma \, \sqrt{-\gamma} \left[\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - \Lambda(p-1)\right] . \tag{38}$$

Here γ_{ab} is the ordinary Riemannian metric on the $p+1$-dimensional brane world-volume with $\gamma \equiv \det ||\gamma_{ab}||$. The world-volume indices $a, b = 0, 1, \ldots, p$; T is the given *ad hoc* brane tension; the constant Λ can be absorbed by rescaling T (see below Eq.(43). The equations of motion w.r.t. γ^{ab} and X^μ read:

$$T_{ab} \equiv \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu\right) G_{\mu\nu} + \gamma_{ab}\frac{\Lambda}{2}(p-1) = 0 \; , \tag{39}$$

$$\partial_a \left(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \sqrt{-\gamma}\gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0 \; . \tag{40}$$

Eqs.(39) when $p \neq 1$ imply:

$$\Lambda \gamma_{ab} = \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} \; , \tag{41}$$

which in turn allows to rewrite Eq.(39) as:

$$T_{ab} \equiv \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{p+1}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu\right) G_{\mu\nu} = 0 \; . \tag{42}$$

Furthermore, using (41) the Polyakov-type brane action (38) becomes on-shell equivalent to the Nambu-Goto-type brane action:

$$S = -T\Lambda^{-\frac{p-1}{2}} \int d^{p+1}\sigma \, \sqrt{-\det ||\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}||} \; . \tag{43}$$

4.1 Modified-Measure Brane Actions

Now, similarly to the string case we introduce a modified world-volume integration measure in terms of $p+1$ auxiliary scalar fields φ^i $(i=1,\ldots,p+1)$:

$$\Phi(\varphi) \equiv \frac{1}{(p+1)!}\varepsilon_{i_1\ldots i_{p+1}}\varepsilon^{a_1\ldots a_{p+1}}\partial_{a_1}\varphi^{i_1}\ldots\partial_{a_{p+1}}\varphi^{i_{p+1}} , \tag{44}$$

and consider the following modified p-brane action:

$$S = -\int d^{p+1}\sigma\, \Phi(\varphi)\Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) + \frac{1}{\sqrt{-\gamma}}\Omega(A)\Big] + \int d^{p+1}\sigma\, \mathcal{L}(A) . \tag{45}$$

The term $\Omega(A)$ indicates a topological density given in terms of some auxiliary gauge (or matter) fields A^I living on the world-volume, "topological" meaning that:

$$\frac{\partial\Omega}{\partial A^I} - \partial_a\left(\frac{\partial\Omega}{\partial\partial_a A^I}\right) = 0 \;\; \text{identically} \quad , \quad \text{i.e. } \delta\Omega(A) = \partial_a\left(\frac{\partial\Omega}{\partial\partial_a A^I}\delta A^I\right) . \tag{46}$$

$\mathcal{L}(A)$ describes possible coupling of the auxiliary fields A^I to external "currents" on the brane world-volume.

The requirement for $\Omega(A)$ to be a topological density is dictated by the requirement that the modified-measure brane action (45) (in the absence of the last gauge/matter term $\int d^{p+1}\sigma\, \mathcal{L}(A)$) reproduces the ordinary p-brane equations of motion apart from the fact that the brane tension $T \equiv \Phi(\varphi)/\sqrt{-\gamma}$ is now an *additional dynamical degree of freedom*.

The simplest example of a topological density $\Omega(A)$ for the auxiliary gauge/matter fields is:

$$\Omega(A) = -\frac{\varepsilon^{a_1\ldots a_{p+1}}}{p+1}F_{a_1\ldots a_{p+1}}(A) \quad , \quad F_{a_1\ldots a_{p+1}}(A) = (p+1)\partial_{[a_1}A_{a_2\ldots a_{p+1}]} , \tag{47}$$

where $A_{a_1\ldots a_p}$ is rank p antisymmetric tensor (Abelian) gauge field on the world-volume[2].

More generally, for $p+1=rs$ we can have:

$$\Omega(A) = \frac{1}{rs}\varepsilon^{a_{11}\ldots a_{1r}\ldots a_{s1}\ldots a_{sr}}F_{a_{11}\ldots a_{1r}}\ldots F_{a_{s1}\ldots a_{sr}} . \tag{48}$$

We may also employ *non-Abelian* auxiliary gauge fields as in the string case. For instance, when $p=3$ we may take:

$$\Omega(A) = \frac{1}{4}\varepsilon^{abcd}\,\mathrm{Tr}\left(F_{ab}(A)F_{cd}(A)\right) \tag{49}$$

or, more generally, for $p+1=2q$:

$$\Omega(A) = \frac{1}{2q}\varepsilon^{a_1b_1\ldots a_qb_q}\,\mathrm{Tr}\left(F_{a_1b_1}\ldots F_{a_qb_q}\right) , \tag{50}$$

where $F_{ab}(A) = \partial_a A_b - \partial_b A_c + i\big[A_a, A_b\big]$.

The modified p-brane action (45) produces the following equations of motion w.r.t. φ^i :

$$\frac{1}{2}\gamma^{cd}\partial_c X^\mu\partial_d X^\nu G_{\mu\nu} + \frac{1}{\sqrt{-\gamma}}\Omega(A) = M \equiv \text{const} , \tag{51}$$

and w.r.t. γ^{ab} (assuming that $\int d^{p+1}\sigma\, \mathcal{L}(A)$ does not depend on γ_{ab} – true *e.g.* if it describes coupling of the auxiliary (gauge) fields A to charged lower-dimensional branes) :

$$\partial_a X^\mu\partial_b X^\nu G_{\mu\nu} + \frac{\gamma_{ab}}{\sqrt{-\gamma}}\Omega(A) = 0 . \tag{52}$$

Both Eqs.(51)–(52) imply:

$$\Omega(A) = -\frac{2M}{p-1}\sqrt{-\gamma} , \tag{53}$$

[2] A modified p-brane model significantly different from (45)–(47) has been proposed in ref.[7]. The latter model also contains world-volume p-form gauge fields which, however, appear quadratically in the brane action of [7] and, therefore, they are dynamical rather than auxiliary fields.

$$\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} = \gamma_{ab}\frac{2M}{p-1} \quad , \quad \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{1}{p+1}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu G_{\mu\nu} = 0 \; . \tag{54}$$

The last two Eqs.(54) reproduce two of the ordinary brane equations of motion (41)–(42) in the standard Polyakov-type formulation.

We now consider the modified brane (45) equations of motion w.r.t. auxiliary (gauge) fields A^I – these are the eqs. determining the dynamical brane tension $T \equiv \Phi(\varphi)/\sqrt{-\gamma}$:

$$\partial_a \Bigl(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\Bigr)\frac{\partial\Omega}{\partial\partial_a A^I} + j_I = 0 \; , \tag{55}$$

where $j_I \equiv \frac{\partial\mathcal{L}}{\partial A^I} - \partial_a\Bigl(\frac{\partial\mathcal{L}}{\partial\partial_a A^I}\Bigr)$ is the corresponding "current" coupled to A^I.

As a physically interesting example let us take the choice (47) for the topological density $\Omega(A)$ and consider the following natural coupling of the auxiliary p-form gauge field:

$$\int d^{p+1}\sigma\, \mathcal{L}(A) = \int d^{p+1}\sigma\, A_{a_1 \ldots a_p} j^{a_1\ldots a_p} \tag{56}$$

to an external world-volume current:

$$j^{a_1\ldots a_p} = \sum_i e_i \int_{\mathcal{B}_i} d^p u\, \frac{1}{p!}\varepsilon^{\alpha_1\ldots\alpha_p}\frac{\partial\sigma_i^{a_1}}{\partial u^{\alpha_1}}\cdots\frac{\partial\sigma_i^{a_p}}{\partial u^{\alpha_p}}\delta^{(p+1)}\bigl(\underline{\sigma} - \underline{\sigma}_i(\underline{u})\bigr) \; . \tag{57}$$

Here $j^{a_1\ldots a_p}$ is a current of charged $(p-1)$-sub-branes $\mathcal{B}_i$ embedded into the original p-brane world-volume via $\sigma^a = \sigma_i^a(\underline{u})$ with parameters $\underline{u} \equiv (u^\alpha)_{\alpha=0,\ldots,p-1}$. For simplicity we assume that the $\mathcal{B}_i$ sub-branes do not intersect each other. With the choice (56)–(57), Eq.(55) for the dynamical brane tension $T \equiv \Phi(\varphi)/\sqrt{-\gamma}$ becomes:

$$\partial_a \Bigl(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\Bigr) + \sum_i e_i \mathcal{N}_a^{(i)} = 0 \; , \tag{58}$$

where $\mathcal{N}_a^{(i)}$ is the normal vector w.r.t. world-hypersurface of the $(p-1)$-sub-brane $\mathcal{B}_i$:

$$\mathcal{N}_a^{(i)} \equiv \frac{1}{p!}\varepsilon_{ab_1\ldots b_p}\int_{\mathcal{B}_i} d^p u\, \frac{1}{p!}\varepsilon^{\alpha_1\ldots\alpha_p}\frac{\partial\sigma_i^{a_1}}{\partial u^{\alpha_1}}\cdots\frac{\partial\sigma_i^{a_p}}{\partial u^{\alpha_p}}\delta^{(p+1)}\bigl(\underline{\sigma} - \underline{\sigma}_i(\underline{u})\bigr) \; . \tag{59}$$

Finally, the modified-brane action (45) yields the X^μ-equations of motion $\partial_a\left(\Phi(\varphi)\gamma^{ab}\partial_b X^\mu\right) = 0$ (taking for simplicity $G_{\mu\nu} = \eta_{\mu\nu}$), which upon using (58) can be rewritten in the form [3] :

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\partial_a\left(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) - \sum_i e_i \mathcal{N}_a^{(i)}\gamma^{ab}\partial_b X^\mu = 0 \; . \tag{60}$$

4.2 Confinement of Charged Lower-Dimensional Branes

Let us consider the solutions for the for the dynamical brane tension Eq.(58). Recalling the definition (59) of $\mathcal{N}_a^{(i)}$ we find from (58) that $T \equiv \Phi(\varphi)/\sqrt{-\gamma}$ is piece-wise constant on the p-brane world-volume with jumps when crossing the world-hypersurface of each charged $(p-1)$-sub-brane $\mathcal{B}_i$, the corresponding jump being equal to the charge magnitude $\pm e_i$ (the overall sign depending on the direction of crossing w.r.t. the normal $\mathcal{N}_a^{(i)}$).

Taking into account the above piece-wise constant solution for $T \equiv \Phi(\varphi)/\sqrt{-\gamma}$, the X^μ-equations of motion (60) for the closed modified brane (45) become equivalent to the following set of equations of motion:

$$\partial_a\left(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) = 0 \quad , \quad \partial_N X^\mu\Bigr|_{\mathcal{B}_i} = 0 \; , \tag{61}$$

[3] For a detailed description of techniques for obtaining solutions of equations of motion for standard string and brane systems in non-trivial backgrounds, which can be easily adapted in the present modified-measure string and brane models, see ref.[8].

where ∂_N indicates normal derivative w.r.t. world-hypersurface of the $(p-1)$-sub-brane $\mathcal{B}_i$. Therefore, Eqs.(61) together with Eq.(58) describe a set of *ordinary open* p-brane segments with common boundaries, where each open p-brane segment possesses *different constant* brane tension and obeys Neumann boundary conditions.

Integrating Eq.(58) along arbitrary smooth closed curve $\mathcal{C}$ on the p-brane world-volume which is transversal to (some or all of) the $(p-1)$-sub-brane $\mathcal{B}_i$, we obtain the following constraints on the possible sub-brane configurations:

$$\sum_i e_i \, n_i(\mathcal{C}) = 0 \; , \tag{62}$$

where $n_i(\mathcal{C})$ is the sign-weighted total number of $\mathcal{C}$ crossing $\mathcal{B}_i$. In the present $p \geq 2$-brane case, however, due to the much more complicated topologies of the pertinent world-volumes Eq.(62) may yield various different types of allowed sub-brane configurations.

As a simple illustration, here we will only consider the simplest non-trivial case $p = 2$ and take the static gauge for the $p = 1$ sub-branes (strings), *i.e.*, the proper times of the charged strings coinsides with the proper time of the bulk membrane. The latter means that the fixed-time world-volume of the bulk *closed* membrane is a Riemann surface with some number g of handles and no holes. Further, we will assume the following simple topology of the attached N charged strings $\mathcal{B}_i$: upon cutting the membrane surface along these attached strings it splits into N *open* membranes $\mathcal{M}_i$ $(i = 1, \ldots, N)$ with Neumann boundary conditions (cf. (61)), each of which being a Riemann surface with g_i handles and 2 holes (boundaries) formed by the strings $\mathcal{B}_{i-1}$ and $\mathcal{B}_i$, respectively[4]. The brane tension of $\mathcal{M}_i$ is a dynamically generated constant T_i where $T_{i+1} = T_i + e_i$. In the present configuration Eq.(62) evidently reduces to the constraint $\sum_i e_i = 0$.

Thus, we conclude that similarly to the string case, modified-measure p-brane models describe configurations of charged $(p-1)$-branes with charge confinement. Apart from the latter, in general there exist more complicated configurations allowed by the constraint (62), which will be studied elsewhere.

5 Conclusions

The above discussion shows that there exist natural from physical point of view modifications of world-sheet and world-volume integration measures which may significantly affect string and brane dynamics. Let us sumarize the main features of the new modified-measure string and brane models:

- Acceptable dynamics *naturally* requires the introduction of auxiliary world-sheet gauge field (world-volume p-form tensor gauge field).
- The string/brane tension is *not* anymore a constant scale given *ad hoc*, but rather appears as an *additional dynamical degree of freedom* beyond the ordinary string/brane degrees of freedom.
- The dynamical string/brane tension has physical meaning of an electric field strenght for the auxiliary gauge field.
- The dynamical string/brane tension obeys "Gauss law" constraint equation and may be non-trivially variable in the presence of point-like charges (on the string world-sheet) or charged lower-dimensional branes (on the p-brane world-volume).
- Modified-measure string/brane models provide simple classical mechanisms for confinement of point-like "color" charges or charged lower-dimensional branes due to variable dynamical tension.

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[4]The Euler charasteristics of the bulk membrane Riemann surface is $\chi = 2 - 2g$, whereas for the open brane $\mathcal{M}_i$ it is $\chi_i = 2 - 2g_i - 2$, so that $\chi = \sum_i \chi_i$ or, equivalently, $g = 1 + \sum_i g_i$.

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